Worksheet on Matrix And Determinant

Summer vacation Homework

1	If a matrix has 8 elements, what are the possible orders it can have? What if it has 5 elements?
2	If $A = [a_{jj}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and $B = [b_{jj}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$ then find $a_{22} + b_{21}$
3	If $A = [a_{jj}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and $B = [b_{jj}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$ then find $a_{11}\, b_{11} + a_{22}\, b_{22}$
4	Construct a 2×3 matrix $A = [a_{jj}]$ whose elements a_{jj} are given by : $a_{ij} = i \times j$
5	Construct a 2×3 matrix $A = [a_{jj}]$ whose elements a_{jj} are given by : $a_{ij} = 2i - j$
5	Construct a 2×3 matrix $A = [a_{jj}]$ whose elements a_{jj} are given by : $a_{ij} = \dfrac{(i+j)^2}{2}$
6	Construct a 2×2 matrix $A = [a_{jj}]$ whose elements a_{jj} are given by : $a_{ij} = e^{2ix} \sin xj$

7	Construct a 3×4 matrix A = $[a_{jj}]$ whose elements a_{jj} are given by : $a_{jj} = \frac{1}{2}\lvert -3i + j\rvert$

8	Find x, y, a and b if $\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$.
9	Find x, y, a and b if $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$
10	Find the values of a, b, c and d from the following equations: $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$
11	Find x, y and z so that A = B, where $A = \begin{bmatrix} x-2 & 3 & 2z \\ 18z & y+2 & 6z \end{bmatrix}, B = \begin{bmatrix} y & z & 6 \\ 6y & z & 2y \end{bmatrix}$
12	If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-\omega \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find x, y, z, ω.
13	If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$ Obtain the values of a, b, c, x, y and z.
14	The sales figure of two car dealers during January 2023 showed that dealer A sold 5 deluxe, 3 premium and 4 standard cars, while dealer B sold 7 deluxe, 2 premium and 3 premium and 4 standard cars, while dealer B sold 7 deluxe, 2 premium and 3 standard cars. Total sales over the 2 month period of January – February revealed that dealer A sold 8 deluxe 7 premium and 6 standard

	cars. In the same 2 month period, dealer B sold 10 deluxe, 5 premium and 7 standard cars. Write 2 x 3 matrices summarizing sales data for January and 2 – month period for each dealer.
15	Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$. Find each of the following : i. 2A - 3B ii. B - 4C iii. 3A - C iv. 3A - 2B + 3C
16	Let $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$. Compute 2A - 3B + 4C.
17	If A = diag (2, -5, 9), B = diag (1, 1, -4) and C = diag (-6, 3, 4), find i. A - 2B ii. B + C - 2A iii. 2A + 3B - 5C
18	Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X - Y \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.
19	If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & \omega \\ 0 & 6 \end{bmatrix}$, then find the values of x, y, z and ω.
20	Find the values of a and b if A = B, where $A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}, B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-10 \end{bmatrix}$

21	Find x, y satisfying the matrix equations $$x\begin{bmatrix}2\\1\end{bmatrix}+y\begin{bmatrix}3\\5\end{bmatrix}+\begin{bmatrix}-8\\-11\end{bmatrix}=O$$
22	Find the value of λ, a non-zero scalar, if $\lambda\begin{bmatrix}1 & 0 & 2\\3 & 4 & 5\end{bmatrix}+2\begin{bmatrix}1 & 2 & 3\\-1 & -3 & 2\end{bmatrix}=\begin{bmatrix}4 & 4 & 10\\4 & 2 & 14\end{bmatrix}$.
23	In a certain city there are 30 colleges. Each college has 15 peons, 6 clerks, 1 typist and 1 section officer. Express the given information as a column matrix. Using scalar multiplication, find the total number of posts of each kind in all the colleges.
24	The monthly incomes of Aryan and Babban are in the ration 3: 4 and their monthly expenditures are in the ratio 5: 7. If each saves 15000 per month, find their monthly incomes using the matrix method. This problem reflects which value?
25	Compute the indicated products: 1. $$\begin{bmatrix}2 & 3 & 4\\3 & 4 & 5\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & -3 & 5\\0 & 2 & 4\\3 & 0 & 5\end{bmatrix}$$
26	Compute the products AB and BA whichever exists in each of the following cases: $$A=\begin{bmatrix}1 & -1 & 2 & 3\end{bmatrix} \text{ and } B=\begin{bmatrix}0\\1\\3\\2\end{bmatrix}$$
27	Compute the products AB and BA whichever exists in each of the following cases: $$\begin{bmatrix}a, b\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}+\begin{bmatrix}a & b & c & d\end{bmatrix}\begin{bmatrix}a\\b\\c\\d\end{bmatrix}$$

28	Evaluate the following: $$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$
29	If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, prove that (A - 2I) (A - 3I) = O.
30	If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, show that $A^2 = O$
31	Compute the elements a_{43} and a_{22} of the matrix: $$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$
32	If ω is a complex cube root of unity, show that $$\left(\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
33	If $\begin{bmatrix} 1 & 1 & x \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$, find x.

34	If $\begin{bmatrix} 1 & -1 & x \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$, find x.
35	If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find k such that $A^2 = kA - 2I_2$.
36	If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = x^2 - 2x - 3$, show that $f(A) = 0$.
37	If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find λ, μ so that $A^2 = \lambda A + \mu I$
38	Solve the matrix equations: $\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix} = 0$
39	If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that A is a root of the polynomial $f(x) = x^3 - 6x^2 + 7x + 2$.

40	If $A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$, find A^{16}.
41	If A and B are square matrices of the same order such that AB = BA, then show that $(A + B)^2 = A^2 + 2AB + B^2$.
42	75. To promote making of toilets for women, an organization tried to generate awareness through (i) house calls (ii) letters, and (iii) announcements. The cost for each mode per attempt is given below : (i) ₹ 50 (ii) ₹ 20 (iii) ₹ 40 The number of attempts made in three villages X, Y and Z are given below :
43	There are 2 families A and B. There are 4 men, 6 women and 2 children in family A, and 2 men, 2 women and 4 children in family B. The recommended daily amount of calories is 2400 for men, 1990 for women, 1800 for children and 45 grams of proteins for men, 55 grams for women and 33 grams for children. Represent the above information using a matrix. Using matrix multiplication, calculate the total requirement of calories and proteins for each of the two families. What awareness can you create among people about the planned diet from this question?
44	The monthly incomes of Aryan and Babbar are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves Rs 15000 per month, find their monthly incomes using matrix method. This problem reflects which value?
45	If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, find x satisfying $0 < x < \frac{\pi}{2}$ when $A + A^T = I$.
46	If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^4 = \lambda A$, then write the value c
47	For any square matrix write whether AA^T is symmetric or skew-symmetric.

48	If matrix $A = \left[a_{ij}\right]_{2\times 2}$, where $a_{ij} = \begin{cases} 1, & \text{if } i \neq j \\ 0, & \text{if } i + j \end{cases}$, then A^2 is equal to
49	If A and B are matrices of the same order, then $AB^T - B^TA$ is a A. skew-symmetric matrix B. null matrix C. unit matrix D. symmetric matrix
50	The matrix $A = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a A. square matrix B. diagonal matrix C. unit matrix D. none of these